Mathematica 11.3 Integration Test Results

Test results for the 85 problems in "6.2.7 hyper^m (a+b cosh^n)^p.m"

Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sinh}[x]^7}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 78 leaves, 4 steps):

$$-\frac{(a+b)^3\,\text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,b^{7/2}}+\frac{\left(a^2+3\,a\,b+3\,b^2\right)\text{Cosh}[x]}{b^3}-\frac{(a+3\,b)\,\text{Cosh}[x]^3}{3\,b^2}+\frac{\text{Cosh}[x]^5}{5\,b}$$

Result (type 3, 148 leaves):

$$-\frac{(a+b)^3\,\text{ArcTan}\left[\frac{\sqrt{b}-i\,\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}\,b^{7/2}}-\frac{(a+b)^3\,\text{ArcTan}\left[\frac{\sqrt{b}+i\,\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}\,b^{7/2}}+\frac{\left(8\,a^2+22\,a\,b+19\,b^2\right)\text{Cosh}[x]}{8\,b^3}-\frac{(4\,a+9\,b)\,\text{Cosh}[3\,x]}{48\,b^2}+\frac{\text{Cosh}[5\,x]}{80\,b}$$

Problem 7: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[x]^5}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$\frac{(a+b)^2\,\text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,b^{5/2}}-\frac{(a+2\,b)\,\text{Cosh}[x]}{b^2}+\frac{\text{Cosh}[x]^3}{3\,b}$$

Result (type 3, 120 leaves):

$$\frac{1}{12\, b^{5/2}}\left(\frac{12\,(a+b)^2\,\text{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}}+\frac{12\,(a+b)^2\,\text{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}}-3\sqrt{b}\,(4a+7b)\,\text{Cosh}[x]+b^{3/2}\,\text{Cosh}[3x]\right)$$

Problem 8: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[x]^3}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 36 leaves, 3 steps):

$$-\frac{(a+b)\,\text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,b^{3/2}}+\frac{\text{Cosh}[x]}{b}$$

Result (type 3, 83 leaves):

$$-\frac{(a+b)\left(\text{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]+\text{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]\right)}{\sqrt{a}\,b^{3/2}}+\frac{\text{Cosh}[x]}{b}$$

Problem 10: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 42 leaves, 4 steps):

$$-\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,(a+b)}-\frac{\text{ArcTanh}[\text{Cosh}[x]]}{a+b}$$

Result (type 3, 106 leaves):

$$-\frac{1}{a+b}\left(\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{b}-i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}}+\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{b}+i\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}}+\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]\right)$$

Problem 11: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]^3}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 61 leaves, 5 steps):

$$\frac{b^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,(a+b)^2}+\frac{(a+3\,b)\,\text{ArcTanh}[\text{Cosh}[x]]}{2\,(a+b)^2}-\frac{\text{Coth}[x]\,\text{Csch}[x]}{2\,(a+b)}$$

Result (type 3, 154 leaves):

$$\frac{1}{8\,\sqrt{a}\,(a+b)^2}\left(8\,b^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{b}-i\,\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]+8\,b^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{b}+i\,\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]-\sqrt{a}\,(a+b)\,\text{Csch}\left[\frac{x}{2}\right]^2+4\,\sqrt{a}\,(a+3\,b)\,\left(\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]\right)-\sqrt{a}\,(a+b)\,\text{Sech}\left[\frac{x}{2}\right]^2\right)$$

Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]^5}{a+b\,\text{Cosh}[x]^2}\,dx$$

Optimal (type 3, 94 leaves, 6 steps):

$$-\frac{b^{5/2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cosh}[x]}{\sqrt{a}}\right]}{\sqrt{a}\,(a+b)^3}-\frac{\left(3\,a^2+10\,a\,b+15\,b^2\right)\,\text{ArcTanh}[\text{Cosh}[x]]}{8\,(a+b)^3}+\frac{(3\,a+7\,b)\,\text{Coth}[x]\,\text{Csch}[x]}{8\,(a+b)^2}-\frac{\text{Coth}[x]\,\text{Csch}[x]^3}{4\,(a+b)}$$

Result (type 3, 229 leaves):

$$\frac{1}{64\,\sqrt{a}\,(a+b)^3}\left(2\,\sqrt{a}\,\left(3\,a^2+10\,a\,b+7\,b^2\right)\,\text{Csch}\left[\frac{x}{2}\right]^2-\sqrt{a}\,(a+b)^2\,\text{Csch}\left[\frac{x}{2}\right]^4-8\left(8\,b^{5/2}\,\text{ArcTan}\left[\frac{\sqrt{b}-i\,\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]+8\,b^{5/2}\,\text{ArcTan}\left[\frac{\sqrt{b}+i\,\sqrt{a+b}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a}}\right]+\sqrt{a}\,\left(3\,a^2+10\,a\,b+15\,b^2\right)\,\left(\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]\right)\right)+2\,\sqrt{a}\,\left(3\,a^2+10\,a\,b+7\,b^2\right)\,\text{Sech}\left[\frac{x}{2}\right]^2+\sqrt{a}\,(a+b)^2\,\text{Sech}\left[\frac{x}{2}\right]^4\right)$$

Problem 56: Result is not expressed in closed-form.

$$\int \frac{1}{a + b\,\mathrm{Cosh}[x]^3}\,dx$$

Optimal (type 3, 288 leaves, 8 steps):

$$\frac{2\,\mathrm{ArcTanh}\left[\frac{\sqrt{a^{1/3}-b^{1/3}}\,\mathrm{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}+b^{1/3}}}\right]}{3\,a^{2/3}\,\sqrt{a^{1/3}-b^{1/3}}\,\sqrt{a^{1/3}+b^{1/3}}}+\frac{2\,\mathrm{ArcTanh}\left[\frac{\sqrt{a^{1/3}+(-1)^{1/3}\,b^{1/3}}\,\mathrm{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}-(-1)^{1/3}\,b^{1/3}}}\right]}{3\,a^{2/3}\,\sqrt{a^{1/3}-\left(-1\right)^{1/3}\,b^{1/3}}\,\sqrt{a^{1/3}+\left(-1\right)^{1/3}\,b^{1/3}}}+\frac{2\,\mathrm{ArcTanh}\left[\frac{\sqrt{a^{1/3}-(-1)^{2/3}\,b^{1/3}}\,\mathrm{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}+(-1)^{2/3}\,b^{1/3}}}\right]}{3\,a^{2/3}\,\sqrt{a^{1/3}-\left(-1\right)^{2/3}\,b^{1/3}}\,\sqrt{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}}}$$

Result (type 7, 105 leaves):

$$\frac{2}{3}\,\mathrm{RootSum}\left[b+3\,b\,\#1^2+8\,a\,\#1^3+3\,b\,\#1^4+b\,\#1^6\,\&,\ \frac{x\,\#1+2\,\mathrm{Log}\left[-\mathrm{Cosh}\left[\frac{x}{2}\right]-\mathrm{Sinh}\left[\frac{x}{2}\right]+\mathrm{Cosh}\left[\frac{x}{2}\right]\#1-\mathrm{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1}{b+4\,a\,\#1+2\,b\,\#1^2+b\,\#1^4}\,\&\right]$$

Problem 57: Result is not expressed in closed-form.

$$\int \frac{1}{a - b\,\mathrm{Cosh}[x]^3}\,dx$$

Optimal (type 3, 288 leaves, 8 steps):

$$\frac{2\,\mathrm{ArcTanh}\left[\frac{\sqrt{a^{1/3}+b^{1/3}}\,\mathrm{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}-b^{1/3}}}\right]}{3\,a^{2/3}\,\sqrt{a^{1/3}-b^{1/3}}\,\sqrt{a^{1/3}+b^{1/3}}}+\frac{2\,\mathrm{ArcTanh}\left[\frac{\sqrt{a^{1/3}-(-1)^{1/3}\,b^{1/3}}\,\mathrm{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}+(-1)^{1/3}\,b^{1/3}}}\right]}{3\,a^{2/3}\,\sqrt{a^{1/3}-\left(-1\right)^{1/3}\,b^{1/3}}\,\sqrt{a^{1/3}+\left(-1\right)^{1/3}\,b^{1/3}}}+\frac{2\,\mathrm{ArcTanh}\left[\frac{\sqrt{a^{1/3}+(-1)^{2/3}\,b^{1/3}}\,\mathrm{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/3}-(-1)^{2/3}\,b^{1/3}}}\right]}{3\,a^{2/3}\,\sqrt{a^{1/3}-\left(-1\right)^{2/3}\,b^{1/3}}\,\sqrt{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}}}$$

Result (type 7, 105 leaves):

$$-\frac{2}{3}\,\mathrm{RootSum}\left[b+3\,b\,\#1^2-8\,a\,\#1^3+3\,b\,\#1^4+b\,\#1^6\,\&,\ \frac{x\,\#1+2\,\mathrm{Log}\left[-\mathrm{Cosh}\left[\frac{x}{2}\right]-\mathrm{Sinh}\left[\frac{x}{2}\right]+\mathrm{Cosh}\left[\frac{x}{2}\right]\#1-\mathrm{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1}{b-4\,a\,\#1+2\,b\,\#1^2+b\,\#1^4}\,\&\right]$$

Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{a+b\,\text{Cosh}[x]^4}\,dx$$

Optimal (type 3, 361 leaves, 10 steps):

$$\frac{\sqrt{\sqrt{a}-\sqrt{a+b}}\;\text{ArcTanh}\left[\frac{\sqrt{\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\,a^{1/4}\,\text{Tanh}[x]}{\sqrt{\sqrt{a}-\sqrt{a+b}}}\right]}{2\sqrt{2}\,a^{3/4}\sqrt{a+b}}-\frac{\sqrt{\sqrt{a}-\sqrt{a+b}}\;\text{ArcTanh}\left[\frac{\sqrt{\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\,a^{1/4}\,\text{Tanh}[x]}{\sqrt{\sqrt{a}-\sqrt{a+b}}}\right]}{2\sqrt{2}\,a^{3/4}\sqrt{a+b}}-\frac{1}{4\sqrt{2}\,a^{3/4}\sqrt{a+b}}\sqrt{\sqrt{a}+\sqrt{a+b}}\;\text{Log}\left[\sqrt{a+b}-\sqrt{2}\,a^{1/4}\sqrt{\sqrt{a}+\sqrt{a+b}}\;\text{Tanh}[x]+\sqrt{a}\;\text{Tanh}[x]^2\right]+\frac{1}{4\sqrt{2}\,a^{3/4}\sqrt{a+b}}\sqrt{\sqrt{a}+\sqrt{a+b}}\;\text{Log}\left[\sqrt{a+b}+\sqrt{2}\,a^{1/4}\sqrt{\sqrt{a}+\sqrt{a+b}}\;\text{Tanh}[x]+\sqrt{a}\;\text{Tanh}[x]^2\right]$$

Result (type 3, 121 leaves):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a}\;\text{Tanh}[x]}{\sqrt{-a+i\sqrt{a}\sqrt{b}}}\right]}{2\sqrt{a}\sqrt{-a+i\sqrt{a}\sqrt{b}}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{a}\;\text{Tanh}[x]}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right]}{2\sqrt{a}\sqrt{a+i\sqrt{a}\sqrt{b}}}$$

Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1+\text{Cosh}[x]^4}\,dx$$

Optimal (type 3, 176 leaves, 10 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{1+\sqrt{2}}-2\,\text{Coth}[x]}{\sqrt{-1+\sqrt{2}}}\right]}{4\sqrt{1+\sqrt{2}}}+\frac{\text{ArcTan}\left[\frac{\sqrt{1+\sqrt{2}}+2\,\text{Coth}[x]}{\sqrt{-1+\sqrt{2}}}\right]}{4\sqrt{1+\sqrt{2}}}-\frac{1}{8}\sqrt{1+\sqrt{2}}\;\text{Log}\left[\sqrt{2}-2\sqrt{1+\sqrt{2}}\;\text{Coth}[x]+2\,\text{Coth}[x]^2\right]+\frac{1}{8}\sqrt{1+\sqrt{2}}\;\text{Log}\left[1+\sqrt{2\left(1+\sqrt{2}\right)}\;\text{Coth}[x]+\sqrt{2}\;\text{Coth}[x]^2\right]$$

Result (type 3, 45 leaves):

$$\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1-i}}\right]}{2\sqrt{1-i}}+\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1+i}}\right]}{2\sqrt{1+i}}$$

Problem 64: Result is not expressed in closed-form.

$$\int \frac{1}{a+b\,\text{Cosh}[x]^5}\,dx$$

Optimal (type 3, 494 leaves, 12 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}-b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-b^{1/5}}\sqrt{a^{1/5}+b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}+(-1)^{1/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{1/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{1/5}b^{1/5}}\sqrt{a^{1/5}+(-1)^{1/5}b^{1/5}}}+$$

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}-(-1)^{2/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{2/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{2/5}b^{1/5}}\sqrt{a^{1/5}+(-1)^{2/5}b^{1/5}}}+$$

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}+(-1)^{3/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{3/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{3/5}b^{1/5}}\sqrt{a^{1/5}+(-1)^{3/5}b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}-(-1)^{4/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{4/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{4/5}b^{1/5}}\sqrt{a^{1/5}+(-1)^{4/5}b^{1/5}}}$$

Result (type 7, 139 leaves):

$$\frac{8}{5}\,\text{RootSum}\Big[b+5\,b\,\#1^2+10\,b\,\#1^4+32\,a\,\#1^5+10\,b\,\#1^6+5\,b\,\#1^8+b\,\#1^{10}\,\&,$$
$$\frac{x\,\#1^3+2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^3}{b+4\,b\,\#1^2+16\,a\,\#1^3+6\,b\,\#1^4+4\,b\,\#1^6+b\,\#1^8}\,\&\Big]$$

Problem 65: Result is not expressed in closed-form.

$$\int \frac{1}{a+b\,\text{Cosh}[x]^6}\,dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}+b^{1/3}}}\right]}{3\,a^{5/6}\sqrt{a^{1/3}+b^{1/3}}}+\frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}-(-1)^{1/3}b^{1/3}}}\right]}{3\,a^{5/6}\sqrt{a^{1/3}-(-1)^{1/3}b^{1/3}}}+\frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}+(-1)^{2/3}b^{1/3}}}\right]}{3\,a^{5/6}\sqrt{a^{1/3}+(-1)^{2/3}b^{1/3}}}$$

Result (type 7, 132 leaves):

$$\frac{16}{3}\,\text{RootSum}\Big[b+6\,b\,\#1+15\,b\,\#1^2+64\,a\,\#1^3+20\,b\,\#1^3+15\,b\,\#1^4+6\,b\,\#1^5+b\,\#1^6\,\&,$$
$$\frac{x\,\#1^2+\text{Log}[-\text{Cosh}[x]-\text{Sinh}[x]+\text{Cosh}[x]\,\#1-\text{Sinh}[x]\,\#1]\,\#1^2}{b+5\,b\,\#1+32\,a\,\#1^2+10\,b\,\#1^2+10\,b\,\#1^3+5\,b\,\#1^4+b\,\#1^5}\,\&\Big]$$

Problem 66: Result is not expressed in closed-form.

$$\int \frac{1}{a + b\, \text{Cosh}[x]^8}\, dx$$

Optimal (type 3, 245 leaves, 9 steps):

$$-\frac{\text{ArcTanh}\left[\frac{(-a)^{1/8}\,\text{Tanh}[x]}{\sqrt{(-a)^{1/4}-b^{1/4}}}\right]}{4\,(-a)^{7/8}\sqrt{(-a)^{1/4}-b^{1/4}}}-\frac{\text{ArcTanh}\left[\frac{(-a)^{1/8}\,\text{Tanh}[x]}{\sqrt{(-a)^{1/4}-i\,b^{1/4}}}\right]}{4\,(-a)^{7/8}\sqrt{(-a)^{1/4}-i\,b^{1/4}}}-\frac{\text{ArcTanh}\left[\frac{(-a)^{1/8}\,\text{Tanh}[x]}{\sqrt{(-a)^{1/4}+i\,b^{1/4}}}\right]}{4\,(-a)^{7/8}\sqrt{(-a)^{1/4}+i\,b^{1/4}}}-\frac{\text{ArcTanh}\left[\frac{(-a)^{1/8}\,\text{Tanh}[x]}{\sqrt{(-a)^{1/4}+b^{1/4}}}\right]}{4\,(-a)^{7/8}\sqrt{(-a)^{1/4}+b^{1/4}}}$$

Result (type 7, 158 leaves):

$$16\,\text{RootSum}\Big[b+8\,b\,\#1+28\,b\,\#1^2+56\,b\,\#1^3+256\,a\,\#1^4+70\,b\,\#1^4+56\,b\,\#1^5+28\,b\,\#1^6+8\,b\,\#1^7+b\,\#1^8\,\&,\ \left(x\,\#1^3+\text{Log}[-\text{Cosh}[x]-\text{Sinh}[x]+\text{Cosh}[x]\,\#1-\text{Sinh}[x]\,\#1]\,\#1^3\right)\Big/\left(b+7\,b\,\#1+21\,b\,\#1^2+128\,a\,\#1^3+35\,b\,\#1^3+35\,b\,\#1^4+21\,b\,\#1^5+7\,b\,\#1^6+b\,\#1^7\right)\,\&\Big]$$

Problem 67: Result is not expressed in closed-form.

$$\int \frac{1}{a - b\, \text{Cosh}[x]^5}\, dx$$

Optimal (type 3, 494 leaves, 12 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}+b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-b^{1/5}}\sqrt{a^{1/5}+b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}-(-1)^{1/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{1/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{1/5}b^{1/5}}\sqrt{a^{1/5}+(-1)^{1/5}b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}+(-1)^{2/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{2/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{2/5}b^{1/5}}\sqrt{a^{1/5}+(-1)^{2/5}b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}-(-1)^{3/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{3/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{3/5}b^{1/5}}\sqrt{a^{1/5}+(-1)^{3/5}b^{1/5}}}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a^{1/5}+(-1)^{4/5}b^{1/5}}\,\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{4/5}b^{1/5}}}\right]}{5\,a^{4/5}\sqrt{a^{1/5}-(-1)^{4/5}b^{1/5}}\sqrt{a^{1/5}+(-1)^{4/5}b^{1/5}}}$$

Result (type 7, 139 leaves):

$$-\frac{8}{5}\,\text{RootSum}\Big[b+5\,b\,\#1^2+10\,b\,\#1^4-32\,a\,\#1^5+10\,b\,\#1^6+5\,b\,\#1^8+b\,\#1^{10}\,\&,\ \frac{x\,\#1^3+2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^3}{b+4\,b\,\#1^2-16\,a\,\#1^3+6\,b\,\#1^4+4\,b\,\#1^6+b\,\#1^8}\,\&\Big]$$

Problem 68: Result is not expressed in closed-form.

$$\int \frac{1}{a - b\,\text{Cosh}[x]^6}\, dx$$

Optimal (type 3, 175 leaves, 7 steps):

$$\frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}-b^{1/3}}}\right]}{3\, a^{5/6}\,\sqrt{a^{1/3}-b^{1/3}}} + \frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}+(-1)^{1/3}\, b^{1/3}}}\right]}{3\, a^{5/6}\,\sqrt{a^{1/3}+(-1)^{1/3}\, b^{1/3}}} + \frac{\text{ArcTanh}\left[\frac{a^{1/6}\,\text{Tanh}[x]}{\sqrt{a^{1/3}-(-1)^{2/3}\, b^{1/3}}}\right]}{3\, a^{5/6}\,\sqrt{a^{1/3}-(-1)^{2/3}\, b^{1/3}}}$$

Result (type 7, 132 leaves):

$$-\frac{16}{3}\,\text{RootSum}\Big[b + 6\, b\, \#1 + 15\, b\, \#1^2 - 64\, a\, \#1^3 + 20\, b\, \#1^3 + 15\, b\, \#1^4 + 6\, b\, \#1^5 + b\, \#1^6\ \&,\ \frac{x\, \#1^2 + \text{Log}[-\text{Cosh}[x] - \text{Sinh}[x] + \text{Cosh}[x]\, \#1 - \text{Sinh}[x]\, \#1]\, \#1^2}{b + 5\, b\, \#1 - 32\, a\, \#1^2 + 10\, b\, \#1^2 + 10\, b\, \#1^3 + 5\, b\, \#1^4 + b\, \#1^5}\ \&\Big]$$

Problem 69: Result is not expressed in closed-form.

$$\int \frac{1}{a - b\,\text{Cosh}[x]^8}\, dx$$

Optimal (type 3, 213 leaves, 9 steps):

$$\frac{\text{ArcTanh}\left[\frac{a^{1/8}\,\text{Tanh}[x]}{\sqrt{a^{1/4}-b^{1/4}}}\right]}{4\, a^{7/8}\,\sqrt{a^{1/4}-b^{1/4}}} + \frac{\text{ArcTanh}\left[\frac{a^{1/8}\,\text{Tanh}[x]}{\sqrt{a^{1/4}-i\, b^{1/4}}}\right]}{4\, a^{7/8}\,\sqrt{a^{1/4}-i\, b^{1/4}}} + \frac{\text{ArcTanh}\left[\frac{a^{1/8}\,\text{Tanh}[x]}{\sqrt{a^{1/4}+i\, b^{1/4}}}\right]}{4\, a^{7/8}\,\sqrt{a^{1/4}+i\, b^{1/4}}} + \frac{\text{ArcTanh}\left[\frac{a^{1/8}\,\text{Tanh}[x]}{\sqrt{a^{1/4}+b^{1/4}}}\right]}{4\, a^{7/8}\,\sqrt{a^{1/4}+b^{1/4}}}$$

Result (type 7, 158 leaves):

$$-16\,\text{RootSum}\Big[b + 8\, b\, \#1 + 28\, b\, \#1^2 + 56\, b\, \#1^3 - 256\, a\, \#1^4 + 70\, b\, \#1^4 + 56\, b\, \#1^5 + 28\, b\, \#1^6 + 8\, b\, \#1^7 + b\, \#1^8\ \&,\ \left(x\, \#1^3 + \text{Log}[-\text{Cosh}[x] - \text{Sinh}[x] + \text{Cosh}[x]\, \#1 - \text{Sinh}[x]\, \#1]\, \#1^3\right) \Big/ \left(b + 7\, b\, \#1 + 21\, b\, \#1^2 - 128\, a\, \#1^3 + 35\, b\, \#1^3 + 35\, b\, \#1^4 + 21\, b\, \#1^5 + 7\, b\, \#1^6 + b\, \#1^7\right)\ \&\Big]$$

Problem 70: Result is not expressed in closed-form.

$$\int \frac{1}{1 + \text{Cosh}[x]^5}\, dx$$

Optimal (type 3, 223 leaves, 11 steps):

$$-\frac{2\,\text{ArcTan}\left[\frac{\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{-\frac{1-(-1)^{1/5}}{1+(-1)^{1/5}}}}\right]}{5\sqrt{-1+(-1)^{2/5}}}-\frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\,\text{ArcTan}\left[\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\left(1+(-1)^{3/5}\right)}+$$
$$\frac{2\,\text{ArcTanh}\left[\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\,\text{ArcTanh}\left[\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\sqrt{1+(-1)^{3/5}}}+\frac{\text{Sinh}[x]}{5\,(1+\text{Cosh}[x])}$$

Result (type 7, 445 leaves):

$$-\frac{1}{10}\text{RootSum}\left[1-2\,\#1+8\,\#1^2-14\,\#1^3+30\,\#1^4-14\,\#1^5+8\,\#1^6-2\,\#1^7+\#1^8\,\&,\right.$$
$$\frac{1}{-1+8\,\#1-21\,\#1^2+60\,\#1^3-35\,\#1^4+24\,\#1^5-7\,\#1^6+4\,\#1^7}$$
$$\left(x+2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]-4\,x\,\#1-\right.$$
$$8\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1+15\,x\,\#1^2+$$
$$30\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^2-40\,x\,\#1^3-$$
$$80\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^3+15\,x\,\#1^4+$$
$$30\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^4-4\,x\,\#1^5-$$
$$8\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^5+x\,\#1^6+$$
$$\left.\left.2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^6\right)\&\right]+\frac{1}{5}\text{Tanh}\left[\frac{x}{2}\right]$$

Problem 72: Result is not expressed in closed-form.

$$\int \frac{1}{1+\text{Cosh}[x]^8}\,dx$$

Optimal (type 3, 129 leaves, 9 steps):

$$\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1-(-1)^{1/4}}}\right]}{4\sqrt{1-(-1)^{1/4}}}+\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1+(-1)^{1/4}}}\right]}{4\sqrt{1+(-1)^{1/4}}}+\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1-(-1)^{3/4}}}\right]}{4\sqrt{1-(-1)^{3/4}}}+\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1+(-1)^{3/4}}}\right]}{4\sqrt{1+(-1)^{3/4}}}$$

Result (type 7, 127 leaves):

$$16\,\text{RootSum}\left[1+8\,\#1+28\,\#1^2+56\,\#1^3+326\,\#1^4+56\,\#1^5+28\,\#1^6+8\,\#1^7+\#1^8\,\&,\right.$$
$$\left.\frac{x\,\#1^3+\text{Log}\left[-\text{Cosh}[x]-\text{Sinh}[x]+\text{Cosh}[x]\,\#1-\text{Sinh}[x]\,\#1\right]\#1^3}{1+7\,\#1+21\,\#1^2+163\,\#1^3+35\,\#1^4+21\,\#1^5+7\,\#1^6+\#1^7}\&\right]$$

Problem 73: Result is not expressed in closed-form.

$$\int \frac{1}{1-\text{Cosh}[x]^5}\,dx$$

Optimal (type 3, 205 leaves, 11 steps):

$$-\frac{2\,\text{ArcTan}\left[\frac{\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right]}{5\sqrt{-1+(-1)^{4/5}}}+\frac{2\,\text{ArcTan}\left[\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\sqrt{-1-(-1)^{3/5}}}+\frac{2\,\text{ArcTanh}\left[\sqrt{\frac{1-(-1)^{1/5}}{1+(-1)^{1/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\,\text{ArcTanh}\left[\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right]}{5\sqrt{1+(-1)^{1/5}}}-\frac{\text{Sinh}[x]}{5\,(1-\text{Cosh}[x])}$$

Result (type 7, 445 leaves):

$$\frac{1}{5}\text{Coth}\left[\frac{x}{2}\right]+\frac{1}{10}\text{RootSum}\Big[1+2\,\#1+8\,\#1^2+14\,\#1^3+30\,\#1^4+14\,\#1^5+8\,\#1^6+2\,\#1^7+\#1^8\,\&,$$
$$\frac{1}{1+8\,\#1+21\,\#1^2+60\,\#1^3+35\,\#1^4+24\,\#1^5+7\,\#1^6+4\,\#1^7}\Big(x+2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]+4\,x\,\#1+$$
$$8\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1+15\,x\,\#1^2+$$
$$30\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^2+40\,x\,\#1^3+$$
$$80\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^3+15\,x\,\#1^4+$$
$$30\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^4+4\,x\,\#1^5+$$
$$8\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^5+x\,\#1^6+$$
$$2\,\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^6\Big)\,\&\Big]$$

Problem 81: Result is not expressed in closed-form.

$$\int \frac{\text{Tanh}[x]^3}{a+b\,\text{Cosh}[x]^3}\,dx$$

Optimal (type 3, 153 leaves, 11 steps):

$$-\frac{b^{2/3}\,\text{ArcTan}\left[\frac{a^{1/3}-2\,b^{1/3}\,\text{Cosh}[x]}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{5/3}}+\frac{\text{Log}[\text{Cosh}[x]]}{a}+\frac{b^{2/3}\,\text{Log}\left[a^{1/3}+b^{1/3}\,\text{Cosh}[x]\right]}{3\,a^{5/3}}-\frac{b^{2/3}\,\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,\text{Cosh}[x]+b^{2/3}\,\text{Cosh}[x]^2\right]}{6\,a^{5/3}}-\frac{\text{Log}\left[a+b\,\text{Cosh}[x]^3\right]}{3\,a}+\frac{\text{Sech}[x]^2}{2\,a}$$

Result (type 7, 145 leaves) :

$$\frac{1}{6\,a}\left(-6\,x+6\,\text{Log}[\text{Cosh}[x]]-2\,\text{RootSum}\Big[b+3\,b\,\#1^2+8\,a\,\#1^3+3\,b\,\#1^4+b\,\#1^6\,\&,\ \left(-b\,x+b\,\text{Log}\left[e^x-\#1\right]-4\,a\,x\,\#1^3+4\,a\,\text{Log}\left[e^x-\#1\right]\#1^3-3\,b\,x\,\#1^4+3\,b\,\text{Log}\left[e^x-\#1\right]\#1^4\right)\Big/\left(b+2\,b\,\#1^2+4\,a\,\#1^3+b\,\#1^4\right)\,\&\Big]+3\,\text{Sech}[x]^2\right)$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Tanh}[x]}{\sqrt{a+b\,\text{Cosh}[x]^3}}\,dx$$

Optimal (type 3, 28 leaves, 4 steps) :

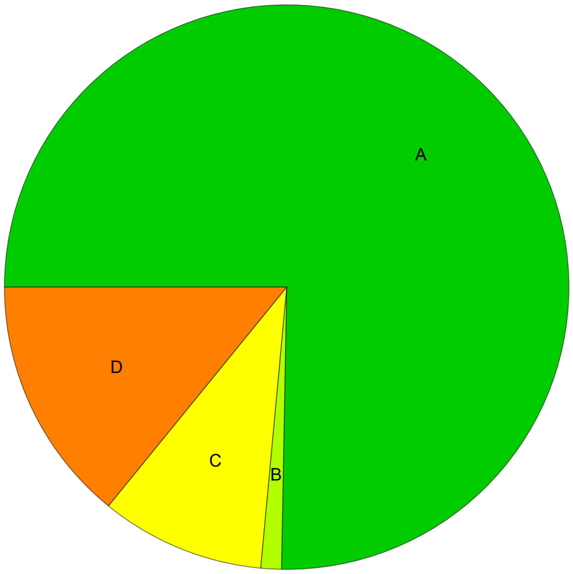
$$-\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cosh}[x]^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}$$

Result (type 3, 66 leaves) :

$$-\frac{2\,\sqrt{b}\,\text{ArcSinh}\left[\frac{\sqrt{a}\,\text{Sech}[x]^{3/2}}{\sqrt{b}}\right]\sqrt{\frac{b+a\,\text{Sech}[x]^3}{b}}}{3\,\sqrt{a}\,\sqrt{a+b\,\text{Cosh}[x]^3}\,\text{Sech}[x]^{3/2}}$$

Summary of Integration Test Results

85 integration problems



A - 64 optimal antiderivatives

B - 1 more than twice size of optimal antiderivatives

C - 8 unnecessarily complex antiderivatives

D - 12 unable to integrate problems

E - 0 integration timeouts